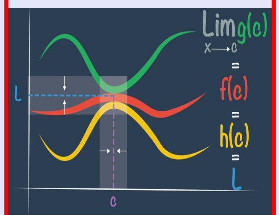
Calculus I

Lecture 12



Feb 19-8:47 AM

Class Quiz 5 Open notes

Box Your Final Ans.

For $\boxed{\varepsilon = .2}$, find $\delta > 0$ such that

$$\lim_{x \to 2} (2x+5) = 9.$$

$f(x) = 2x+5$

$a = 2, \quad L = 9$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|2x+5-9| < .2$ 〃 $|x-2| < \delta$

$|2x-4| < .2$

$|2(x-2)| < .2$

$2|x-2| < .2$

$|x-2| < \frac{.2}{2}$

$\rightarrow |x-2| < .1$

$\boxed{\delta = .1}$

Sep 16-7:11 AM

Prove $\lim_{x \to 4} x^3 = 64$

$f(x) = x^3$

$a = 4$

$L = 64$ ✓

For $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-4| < \delta$

$|x^3 - 64| < \varepsilon$ " "

$|(x-4)(x^2+4x+16)| < \varepsilon$

$\underbrace{|x^2+4x+16|}_{\text{Bound}} \underbrace{|x-4|}_{\text{Keep}} < \varepsilon$

If $|x^2+4x+16| < C$, then $|x^2+4x+16||x-4| < C|x-4| < \varepsilon$

Since $f(x) = x^3$ is a polynomial function, $|x-4| < \frac{\varepsilon}{C}$

we pick $\boxed{\delta \le 1}$

$|x-4| < 1$

$-1 < x-4 < 1$

$3 < x < 5$

$x = 5 \to x^2+4x+16 = 5^2+4(5)+16 = 61$

$x = 3 \to x^2+4x+16 = 3^2+4(3)+16 = 37$

$37 < x^2+4x+16 < 61$

$|x^2+4x+16| < 61$

$C = 61$

So $\delta = \min\left\{1, \frac{\varepsilon}{C}\right\}$

$\delta = \min\left\{1, \frac{\varepsilon}{61}\right\}$

If $\varepsilon = 1 \to \delta = \frac{1}{61}$

If $\varepsilon = 62 \to \delta = 1$

Sep 16-7:40 AM

Prove $\lim_{x \to 2} x^3 = 8$

$f(x) = x^3$

$a = 2$

$L = 8$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|x^3 - 8| < \varepsilon$ " $|x-2| < \delta$

$|(x^2+2x+4)(x-2)| < \varepsilon$

$\underbrace{|x^2+2x+4|}_{\text{Bound}} \underbrace{|x-2|}_{\text{Keep}} < \varepsilon$

If $|x^2+2x+4| < C$, then $C|x-2| < \varepsilon$ $|x-2| < \frac{\varepsilon}{C}$

For $\boxed{\delta \le 1}$, $|x-2| < 1$ $-1 < x-2 < 1$

$1 < x < 3$

If $1 < x < 3$, then $7 < x^2+2x+4 < 19 \to |x^2+2x+4| < 19$

$C = 19$

$\delta = \min\left\{1, \frac{\varepsilon}{19}\right\}$

If $\varepsilon = .1 \to \delta = \frac{1}{190}$

If $\varepsilon = 10 \to \delta = \frac{10}{19}$

If $\varepsilon = 20 \to \delta = 1$

Sep 16-7:51 AM

Prove $\lim\limits_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$f(x) = \frac{1}{x}$

$a = \frac{1}{2}$

$L = 2$ ✓

For every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

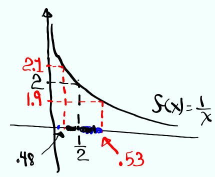
$\left| \frac{1}{x} - 2 \right| < \varepsilon$ whenever $\left| x - \frac{1}{2} \right| < \delta$

$\left| \frac{1 - 2x}{x} \right| < \varepsilon$

$\left| \frac{-2\left(x - \frac{1}{2}\right)}{x} \right| < \varepsilon$

$\frac{2}{|x|} \left| x - \frac{1}{2} \right| < \varepsilon$

Bound   Keep



If $\varepsilon = .1$

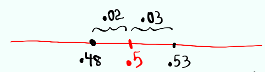
$f(x) = 2.1 \quad \frac{1}{x} = 2.1 \quad x = \frac{1}{2.1} \quad x \approx .48 = .5 - .02$

$f(x) = 1.9 \quad \frac{1}{x} = 1.9 \quad x = \frac{1}{1.9} \quad x = .53 = .5 + .03$

If $\frac{2}{|x|} < C$, then

$C\left| x - \frac{1}{2} \right| < \varepsilon$

$\left| x - \frac{1}{2} \right| < \frac{\varepsilon}{C}$



$\delta = \min\{.02, .03\} = \boxed{.02}$

$\frac{1}{80} = .0125$

Sep 12-8:29 AM

as $x \to \frac{1}{2}$, Pick $\delta \le \frac{1}{4}$

$\left| x - \frac{1}{2} \right| < \frac{1}{4}$

$-\frac{1}{4} < x - \frac{1}{2} < \frac{1}{4}$

Add $\frac{1}{2}$

$\frac{1}{4} < x < \frac{3}{4}$

$4 > \frac{1}{x} > \frac{4}{3} \to \frac{4}{3} < \frac{1}{x} < 4$

$\frac{4}{3} < \left| \frac{1}{x} \right| < 4$

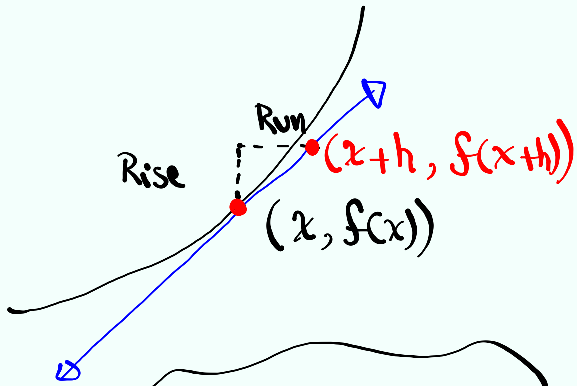
So $\boxed{\delta = \min\left\{\frac{1}{4}, \frac{\varepsilon}{8}\right\}}$

$\frac{2}{|x|} < \underset{\uparrow C}{8}$

If $\varepsilon = .1$

$\delta = \min\left\{\frac{1}{4}, \frac{1}{80}\right\} = \frac{1}{80}$

Sep 16-8:11 AM



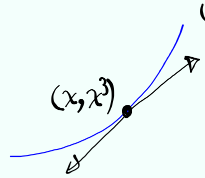
$$m = \frac{\Delta y}{\Delta x}$$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ is the slope of the tangent line at any point to the graph $y = f(x)$.

Sep 16-8:20 AM

Find slope of tan. line to the graph of $f(x) = x^3$ at any point.



$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- x^3}}{h}$$

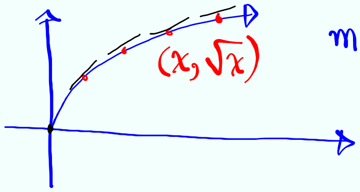
$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim_{h \to 0} (3x^2 + 3x\cancel{h}^{\to 0} + \cancel{h^2}^{\to 0}) = \boxed{3x^2}$$

Slope of tan. line at (2,8) $m = 3(2)^2 = 12$

Slope of tan. line at $x = -1 \Rightarrow m = 3(-1)^2 = \boxed{3}$

Sep 16-8:24 AM

Find slope of the tan. line to the graph of $f(x) = \sqrt{x}$ at any Point.



$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

No slope at $x = 0$

m at $x = 4$

$$m = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

m at $x = 9$

$$m = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

Sep 16-8:29 AM